

# Gravitational mass in electromagnetic field

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A fraction of energy is theoretically predicted to be captured from electromagnetic field to form a gravitating mass, when a low-mass charged particle enters the strong field from a region of no electromagnetism. In this paper the mass variation has been calculated for a charged particle on free-fall in the constraint electromagnetic field. It has been shown that there is an evident effect to the variation in mass when the low-mass charged particle is in the strong electromagnetic field.

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## I. INTRODUCTION

The equality of the inertial and gravitational masses is being doubted all the time. And the equality of masses remains as puzzling as ever. However, the existing theories do not explain why the gravitational mass has to equal the inertial mass, and then do not offer compelling reason for this empirical fact. The paper attempts to reason out why there exists the equality of masses in most cases, even in the electromagnetic field.

Some experiments for the equality of masses have been performed by L. Eötvös [1], H. Potter [2], R. Dicke [3], and V. Braginsky [4], etc. And more precise experiments have been carried out by I. Shapiro [5], K. Nordtvedt [6], and J. Gundlach [7] etc. Presently, no deviation from this equality has ever been found, at least to the accuracy  $10^{-15}$ . But all of these verifications are solely constrained to be in the range of weak gravitational strength, and have not been validated in the strong gravity nor in the electromagnetic field. So this puzzle of the equality of masses remains unclear and has not satisfied results.

The paper brings forward a theoretical model to study the variation of gravitational mass in the electromagnetic field. It carries out the result that the gravitational mass will be varied in the strong strength by capture or release the energy density of the electromagnetic field.

## II. ELECTROMAGNETIC FIELD

The electromagnetic field can be described with the quaternion [8]. In the treatise on electromagnetic field theory, the quaternion was first used by J. Maxwell [9] to demonstrate the electromagnetic field in 1873. And the gravitational field can be described by the quaternion also, and can be used to work out the variation of the gravitational mass in the gravitational field [10].

The gravitational field and electromagnetic field both can be illustrated by the quaternion, but they are quite different from each other indeed. It is assumed for simplicity that there exists one kind of symmetry between the electromagnetic field with the gravitational field. We add another four-dimensional basis vector to the ordinary four-dimensional basis vector to include the feature of the gravitational and electromagnetic fields [11].

Some physical quantities are the functions of coordinates  $r_0, r_1, r_2$ , and  $r_3$  in these two kinds of fields, but the basis vector,  $\mathbb{E}_g = (1, \dot{\mathbf{i}}_1, \dot{\mathbf{i}}_2, \dot{\mathbf{i}}_3)$ , of the gravitational field differs from the basis vector,  $\mathbb{E}_e = (\mathbf{e}, \dot{\mathbf{j}}_1, \dot{\mathbf{j}}_2, \dot{\mathbf{j}}_3)$ , of the electromagnetic field. Two kinds of basis vectors constitute the basis vector  $\mathbb{E} = \mathbb{E}_g + \mathbb{E}_e$  of the octonion [12], with  $\mathbb{E}_e = \mathbb{E}_g \circ \mathbf{e}$ . The symbol  $\circ$  denotes the octonion multiplication. And the octonion basis vectors satisfy the multiplication characteristics in Table I.

In the quaternion space, the radius vector  $\mathbb{R}$  is

$$\mathbb{R} = r_0 + r_1 \dot{\mathbf{i}}_1 + r_2 \dot{\mathbf{i}}_2 + r_3 \dot{\mathbf{i}}_3 \quad (1)$$

where,  $r_0 = ct$ ;  $c$  is the light's speed;  $t$  denotes the time.

The velocity  $\mathbb{V}$  is defined as

$$\mathbb{V} = \partial \mathbb{R} / \partial t = c + v_1 \dot{\mathbf{i}}_1 + v_2 \dot{\mathbf{i}}_2 + v_3 \dot{\mathbf{i}}_3 \quad (2)$$

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TABLE I: The octonion multiplication table.

	1	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{e}$	$\mathbf{j}_1$	$\mathbf{j}_2$	$\mathbf{j}_3$
1	1	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{e}$	$\mathbf{j}_1$	$\mathbf{j}_2$	$\mathbf{j}_3$
$\mathbf{i}_1$	$\mathbf{i}_1$	-1	$\mathbf{i}_3$	$-\mathbf{i}_2$	$\mathbf{j}_1$	$-\mathbf{e}$	$-\mathbf{j}_3$	$\mathbf{j}_2$
$\mathbf{i}_2$	$\mathbf{i}_2$	$-\mathbf{i}_3$	-1	$\mathbf{i}_1$	$\mathbf{j}_2$	$\mathbf{j}_3$	$-\mathbf{e}$	$-\mathbf{j}_1$
$\mathbf{i}_3$	$\mathbf{i}_3$	$\mathbf{i}_2$	$-\mathbf{i}_1$	-1	$\mathbf{j}_3$	$-\mathbf{j}_2$	$\mathbf{j}_1$	$-\mathbf{e}$
$\mathbf{e}$	$\mathbf{e}$	$-\mathbf{j}_1$	$-\mathbf{j}_2$	$-\mathbf{j}_3$	-1	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$
$\mathbf{j}_1$	$\mathbf{j}_1$	$\mathbf{e}$	$-\mathbf{j}_3$	$\mathbf{j}_2$	$-\mathbf{i}_1$	-1	$-\mathbf{i}_3$	$\mathbf{i}_2$
$\mathbf{j}_2$	$\mathbf{j}_2$	$\mathbf{j}_3$	$\mathbf{e}$	$-\mathbf{j}_1$	$-\mathbf{i}_2$	$\mathbf{i}_3$	-1	$-\mathbf{i}_1$
$\mathbf{j}_3$	$\mathbf{j}_3$	$-\mathbf{j}_2$	$\mathbf{j}_1$	$\mathbf{e}$	$-\mathbf{i}_3$	$-\mathbf{i}_2$	$\mathbf{i}_1$	-1

The gravitational potential is  $\mathbb{A}_g = (a_0, a_1, a_2, a_3)$ , and the electromagnetic potential is  $\mathbb{A}_e = (A_0, A_1, A_2, A_3)$ . The gravitational field and electromagnetic field constitute the modified gravity field, which potential is  $\mathbb{A}$ .

$$\mathbb{A} = \mathbb{A}_g + k_{eg}\mathbb{A}_e \quad (3)$$

where,  $k_{eg}$  is the coefficient.

The strength  $\mathbb{B}$  consists of the gravitational strength  $\mathbb{B}_g$  and the electromagnetic strength  $\mathbb{B}_e$ .

$$\mathbb{B} = \diamond \circ \mathbb{A} = \mathbb{B}_g + k_{eg}\mathbb{B}_e \quad (4)$$

where, the operator  $\diamond = \partial_0 + \mathbf{i}_1\partial_1 + \mathbf{i}_2\partial_2 + \mathbf{i}_3\partial_3$ ; and the  $\partial_i = \partial/\partial r_i$ ,  $i = 0, 1, 2, 3$ .

In the above equation, we choose the following gauge conditions to simplify succeeding calculation.

$$\partial_0 a_0 + \nabla \cdot \mathbf{a} = 0, \quad \partial_0 A_0 + \nabla \cdot \mathbf{A} = 0 \quad (5)$$

where, the operator  $\nabla = \mathbf{i}_1\partial_1 + \mathbf{i}_2\partial_2 + \mathbf{i}_3\partial_3$ ;  $\mathbf{a} = \mathbf{i}_1 a_1 + \mathbf{i}_2 a_2 + \mathbf{i}_3 a_3$ ;  $\mathbf{A} = \mathbf{i}_1 A_1 + \mathbf{i}_2 A_2 + \mathbf{i}_3 A_3$ .

The gravitational strength  $\mathbb{B}_g$  includes two components,  $\mathbf{g} = (g_{01}, g_{02}, g_{03})$  and  $\mathbf{b} = (g_{23}, g_{31}, g_{12})$ ,

$$\begin{aligned} \mathbf{g}/c &= \mathbf{i}_1(\partial_0 a_1 + \partial_1 a_0) + \mathbf{i}_2(\partial_0 a_2 + \partial_2 a_0) + \mathbf{i}_3(\partial_0 a_3 + \partial_3 a_0) \\ \mathbf{b} &= \mathbf{i}_1(\partial_2 a_3 - \partial_3 a_2) + \mathbf{i}_2(\partial_3 a_1 - \partial_1 a_3) + \mathbf{i}_3(\partial_1 a_2 - \partial_2 a_1) \end{aligned}$$

meanwhile the electromagnetic strength  $\mathbb{B}_e$  involves two parts,  $\mathbf{E} = (B_{01}, B_{02}, B_{03})$  and  $\mathbf{B} = (B_{23}, B_{31}, B_{12})$ .

$$\begin{aligned} \mathbf{E}/c &= \mathbf{j}_1(\partial_0 A_1 + \partial_1 A_0) + \mathbf{j}_2(\partial_0 A_2 + \partial_2 A_0) + \mathbf{j}_3(\partial_0 A_3 + \partial_3 A_0) \\ \mathbf{B} &= \mathbf{j}_1(\partial_3 A_2 - \partial_2 A_3) + \mathbf{j}_2(\partial_1 A_3 - \partial_3 A_1) + \mathbf{j}_3(\partial_2 A_1 - \partial_1 A_2) \end{aligned}$$

The linear momentum  $\mathbb{S}_g = m\mathbb{V}$  is the source of the gravitational field, and the electric current  $\mathbb{S}_e = q\mathbb{V} \circ \mathbf{e}$  is the source of the electromagnetic field. The source  $\mathbb{S}$  satisfies,

$$\mu\mathbb{S} = -(\mathbb{B}/c + \diamond)^* \circ \mathbb{B} = \mu_g\mathbb{S}_g + k_{eg}\mu_e\mathbb{S}_e - \mathbb{B}^* \circ \mathbb{B}/c \quad (6)$$

where,  $m$  is the mass;  $q$  is the electric charge;  $\mu$ ,  $\mu_g$ , and  $\mu_e$  are the constants;  $k_{eg}^2 = \mu_g/\mu_e$ ;  $*$  denotes the conjugate of the octonion;  $\diamond^2 = \diamond^* \circ \diamond$ .

The  $\mathbb{B}^* \circ \mathbb{B}/(2\mu_g)$  is the energy density, and includes that of the electromagnetic field.

$$\mathbb{B}^* \circ \mathbb{B}/\mu_g = \mathbb{B}_g^* \circ \mathbb{B}_g/\mu_g + \mathbb{B}_e^* \circ \mathbb{B}_e/\mu_e \quad (7)$$

The applied force  $\mathbb{F}$  is defined from the linear momentum  $\mathbb{P} = \mu\mathbb{S}/\mu_g$ , which is the extension of the  $\mathbb{S}_g$ .

$$\mathbb{F} = c(\mathbb{B}/c + \diamond)^* \circ \mathbb{P} \quad (8)$$

where, the applied force  $\mathbb{F}$  includes the gravity, inertial force, interacting force between the angular momentum with gravity, Lorentz force, and interacting force between the magnetic moment with electromagnetic field, etc.

The above means the field equations of the gravity are modified, but Maxwell's equations of the electromagnetic field remind unchanged. Meanwhile the definitions of the applied force and linear momentum etc. are expanded in the gravitational and electromagnetic fields.

In the theoretical model which consists of the electromagnetic and gravitational fields, the low-mass charged particle is on equilibrium state, when the total applied force is equal to zero. The equilibrium equations deduce the free-fall motion of the low-mass charged particle in Figure 1.

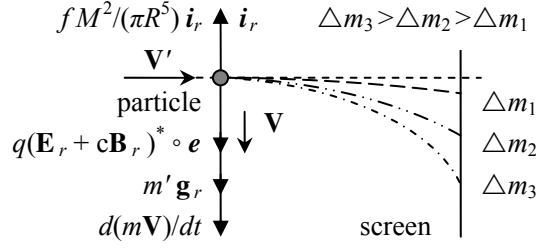


FIG. 1: In the constraint electromagnetic field which strength  $\mathbf{E}_r + c\mathbf{B}_r = 0$ , the low-mass charged particle falls freely along the  $\mathbf{i}_r$  and moves uniformly along the horizontal direction, with  $\Delta m$  being  $\Delta m_1$ ,  $\Delta m_2$ , and  $\Delta m_3$ .

### III. GRAVITATIONAL MASS

Considering the free-fall motion of a low-mass charged particle along the earth's radial direction  $\mathbf{i}_r$  in the electromagnetic field and gravitational field of the earth. We choose the cylindrical polar coordinate system  $(t, r, \theta, z)$  with its origin at the center of the mass of the earth. The z-axis is directed along the rotation axis of the earth, and the  $(r, \theta)$  plane is constrained to coincide with the equatorial plane of the earth. The radius vector  $\mathbb{R}$  and velocity  $\mathbb{V}$  of the particle are respectively,

$$\mathbb{R} = r_0 + \mathbf{r}, \mathbf{r} = r\mathbf{i}_r; \mathbb{V} = c + \mathbf{V}, \mathbf{V} = V_r\mathbf{i}_r. \quad (9)$$

and the strength exerted on the low-mass charged particle is

$$\begin{aligned} \mathbb{B} &= \mathbf{g}/c + k_{eg}\mathbf{E}/c + k_{eg}\mathbf{B} \cdot \mathbf{g} = \mathbf{g}_r, \quad \mathbf{g}_r = g_r\mathbf{i}_r; \\ \mathbf{E} &= \mathbf{E}_r, \quad \mathbf{E}_r = E_r\mathbf{i}_r \circ \mathbf{e}; \quad \mathbf{B} = \mathbf{B}_r, \quad \mathbf{B}_r = B_r\mathbf{i}_r \circ \mathbf{e}. \end{aligned}$$

where,  $\mathbf{i}_r$  is the unitary basis vector.

By Eq.(8), the equilibrium equation of total applied force can be written as

$$(\mathbf{g}_r/c^2 + k_{eg}\mathbf{E}_r/c^2 + k_{eg}\mathbf{B}_r/c + \diamond)^* \circ [m'c + m\mathbf{V} + q(k_{eg}\mu_e/\mu_g)(c + \mathbf{V}) \circ \mathbf{e}] = 0 \quad (10)$$

where,  $m' = m + \Delta m$ ,  $\Delta m = -\mathbb{B}^* \circ \mathbb{B}/(\mu_g c^2)$ ;  $\diamond = \partial/\partial r_0 + \mathbf{i}_r \partial/\partial r$ .

In the cylindrical polar coordinates  $(t, r, \theta, z)$ , the above can be rearranged according to the gravitational field. In the basis vector  $(1, \mathbf{i}_r, \mathbf{i}_\theta, \mathbf{i}_z)$ ,

$$(\mathbf{g}_r/c^2 + \diamond)^* \circ (m'c + m\mathbf{V}) + (\mathbf{E}_r/c^2 + \mathbf{B}_r/c)^* \circ [q(c + \mathbf{V}) \circ \mathbf{e}] = 0 \quad (11)$$

where,  $\mathbf{g}_r^* = -\mathbf{g}_r$ ;  $\mathbf{E}_r^* = -\mathbf{E}_r$ ;  $\mathbf{B}_r^* = -\mathbf{B}_r$ .

It is useful to decompose Eq.(11) along the  $\mathbf{i}_r$ ,

$$\partial(m\mathbf{V})/\partial t - m'\mathbf{g}_r - q(\mathbf{E}_r + c\mathbf{B}_r) \circ \mathbf{e} + fM^2/(\pi R^5)\mathbf{i}_r = 0 \quad (12)$$

where,  $M$  is the mass of the earth, and  $f$  is the universal gravitational constant.

In the paper, if we consider the  $\partial(m\mathbf{V})/\partial t$  and  $m'\mathbf{g}_r$  as the force of inertia and gravity respectively in Eq.(12), then the  $m$  and  $m'$  will be defined as the inertial mass and gravitational mass correspondingly. And their difference is the  $\Delta m$ , which comes from the energy density of the gravitational and electromagnetic fields.

$$-\Delta m = fM^2/(4\pi c^2 R^4) + E_r^2/(c^4 \mu_e) + B_r^2/(c^2 \mu_e) \quad (13)$$

where,  $R$  is the particle's distance to the earth's center.

When it satisfies the condition,  $\mathbf{E}_r + c\mathbf{B}_r = 0$ , the low-mass charged particle is on free-fall. And we can observe the evident effect of the variation of gravitational mass in the fairly strong strength of the electromagnetic field, which may be generated by the laser [13].

$$d(m\mathbf{V})/dt - m'\mathbf{g}_r + fM^2/(\pi R^5)\mathbf{i}_r = 0 \quad (14)$$

where,  $\partial(m\mathbf{V})/\partial t$  is written as  $d(m\mathbf{V})/dt$ , when the time  $t$  is the sole variable;  $m$ ,  $q$ , and  $M$  are the constants;  $\mathbf{E}_r$  and  $\mathbf{B}_r$  both are static and uniform.

Similarly, when the particle's charge  $q = 0$ , the low-mass charged particle is on free-fall as well. And Eq.(14) can be deduced from Eq.(11) directly. It asserts the  $\Delta m$  of the uncharged particle (the neutron etc.) will change with the strength of the electromagnetic field also.

From the above, we find the  $\Delta m$  has a limited effect on the motion of the large-mass particle, because the  $\Delta m$  is quite small. Therefore the equality of masses is believed to be correct in most cases. However, when there exists a very strong strength of electromagnetic field, the  $\Delta m$  can become very huge, and then has an impact on the motion of the low-mass charged particle obviously.

#### IV. CONCLUSIONS

In the theoretical model, the gravitational mass changes with the electromagnetic strength also, and has a small deviation from the inertial mass. This states that it will violate the equality of masses in the fairly strong strength of the electromagnetic field.

The definition of the gravitational mass equates the gravitational mass  $m'$  with the sum of the inertial mass  $m$  and the variation of gravitational mass  $\Delta m$ . The  $\Delta m$  is related to the energy density of gravitational field and electromagnetic field. This inference means that the strong strength of the electromagnetic field will cause the distinct variation of gravitational mass also. But this problem has never been discussed before. In other words, the equality of masses has not been validated in the strong electromagnetic field.

In contrast to the gravitational field, the super-strong strength of electromagnetic field may be easier to achieve. If we could obtain the super-strong electromagnetic field in the lab, more experiments about the equality of masses would be performed. One international research group found that the variation of the proton-electron mass ratio based on lab measurement [14]. It is expected to find the similar experimental results under different strength of the electromagnetic field or gravitational field.

It should be noted that the study for the gravitational mass has examined only one kind of the simple case, of which the low-mass charged particle is on the free-fall motion in the constraint electromagnetic field or the gravitational field. Despite its preliminary character, this study can clearly indicate that the gravitational mass is always changed with the electromagnetic strength. For the future studies, the theoretical model will concentrate on only the suitable predictions about the large variation of gravitational mass in the fairly strong strength of the electromagnetic field.

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